Eur. Phys. J. D **37**, 267–273 (2006)
DOI: 10.1140/epjd/e2005-00274-6

THE EUROPEAN PHYSICAL JOURNAL D

Distributed entanglement in the two-photon two-mode non-linear process

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Received 25 March 2005 / Received in final form 19 September 2005
Published online 11 October 2005 –

Abstract. In this paper we consider a quantum optics model where two-mode quantum light cavity with Kerr-like medium is coupled to an atom via two-photon process. The dynamical evolution of the system is studied in terms of entanglement measured by quantum relative entropy. The entanglements for the different bipartite partitions of the system, i.e., atom-two modes, mode-mode, mode-(atom+mode), are calculated explicitly and interesting trade-off relations between the different kinds of entanglement can be observed in different cases. The results show the entanglement between mode-mode is generally out of phase with that between atom and two modes, even though the two modes do not interact directly, and the Kerr-like medium prevents the atom and two modes from entangling.

PACS. 03.65.Ud Entanglement and quantum nonlocality (e.g. EPR paradox, Bell's inequalities, GHZ states, etc.) – 42.50.-p Quantum optics – 42.50.Dv Nonclassical states of the electromagnetic field, including entangled photon states; quantum state engineering and measurements

1 Introduction

Quantum entanglement, as a physical, is widely used in quantum information processing [1]. In such physically processing one usually needs to find the entanglement properties and the way to control it, therefore studying the dynamic properties of entanglement is useful for processing quantum information. Recently entanglement in many-body physical systems has been vastly studied, and It has become clear that entanglement plays an important role in the understanding of critical and thermodynamical properties of quantum many-body systems [2]. Unlike classical correlations, quantum entanglement cannot be freely shared among many objects. For example, given a triple of spin-1/2 particles A, B, and C, if particles A and B are fully entangled in the state $(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$, then particle A cannot be simultaneously entangled with particle C.

In investigating entanglement in many-body systems, one of the issues is entanglement distribution amongst subsystems, and the involved work was first reported by Coffman, Kundu, and Wootters [3]. By investigating entanglement in tripartite systems ABC with a Hilbert space structure $2 \otimes 2 \otimes 2$, they found that, the squared concurrence (a measure of entanglement for a two-qubit system [4]) between A and B, plus the squared concurrence between A and C, cannot be greater than the squared concurrence between A and the pair BC. Now it has become clear that the limitations on the distribution of quantum entanglement are known as monogamy constraints [5]. While a great deal of progress has been made in understanding special situations [5–7], these constraints have been difficult to quantify in the general cases where the subsystems are continuous (for example, multimode Gaussian states [7]) or infinitely dimensional. This is because that the measure of entanglement has not been completely resolved and is hard to compute. Therefore, in some cases one resorts to concrete physical models, such as spin chain systems [8] and quantum optics systems [9], to research this problem. In reference [9] Tessier et al. investigated the entanglement sharing in the two-atom Tavis-Cummings model by numerical calculation, where the whole system constitutes a tripartite quantum system with Hilbert space structure $2 \otimes 2 \otimes \infty$. In this paper we investigate the entangle dynamics and entanglement distribution in a two-photon two-mode nonlinear Jaynes-Cummings model. Different from the previous model, our system, in a Hilbert space with tensor product structure $2 \otimes \infty \otimes \infty$, involves two modes of cavity field and a two-level atom surrounded by a nonlinear Kerr-like medium in the cavity. Kerr-like medium can be useful in many aspects, such as detection of nonclassical states [10], quantum nondemolition measurement [11], investigation of quantum fluctuations [12], generation of entangled macroscopic quantum states [13], and quantum

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information processing [14,15]. In particular, the influence of Kerr-like medium on the entanglement in a quantum optics system was considered [16], however those studies are limited to the entanglement between a atom and a single mode field. Here we investigate the influence of a Kerr-like medium on the distribution of entanglements among different bipartite subsystems. The merit of our paper is that the amount of entanglement can be computed in terms of quantum relative entropy [17], therefore one can clearly see the entanglement distribution among different subsystems.

The remainder of this paper is organized as follows. The model is given in Section 2. The third part gives the our main results, that is, the entanglement distribution between subsystems. The paper ends in Section 4 with a conclusion.

2 Two-photon two-mode nonlinear Jaynes-Cummings model and time evolution for system

Our system is as follows. An effective two-level atom, whose exited level is $|e\rangle$ with energy E_e and ground level $|g\rangle$ with energy E_g, interacts with two modes (with frequencies ω_1 and ω_2) of quantum light field inside a nonlinear Kerr-like medium. In two-photon processes there are more than two levels involved, but it is possible to neglect them if we assume the condition $\omega_1 + \omega_2 = E_e - E_g (\hbar = 1)$ is satisfied, and we consider the transition frequencies between $|e\rangle$, $|g\rangle$ and the intermediate levels are different from the frequencies of the modes. The Kerr-like medium is modeled as an anharmonic oscillator coupled to the two-mode cavity. Consequently, the effective Hamiltonian, in the adiabatic and rotating wave approximation, reads [18]

$$\begin{aligned} H = {} & (g a_1 a_2 |e\rangle\langle g| + H.c.) \\ & + \left(\chi_1 a_1^{\dagger 2} a_1^2 + \chi_2 a_2^{\dagger 2} a_2^2 + 2\sqrt{\chi_1\chi_2} a_2^\dagger a_1^\dagger a_2 a_1\right), \quad (1) \end{aligned}$$

where a_1 ($a_1^\dagger$) and a_2 ($a_2^\dagger$) are annihilation (creation) operators of the two modes, g is the coupling coefficient between the atomic levels and the two-mode field, here it is a constant consider since the atom is trapped in the cavity field, and $\chi_i (i = 1, 2)$ denotes the dispersive part of the third-order nonlinearity of the Kerr-like medium. The fist part in equation (1) denotes the interaction between the atom and the two-mode field, the second part with three non-linear terms represents the nonlinearly coupling between the two modes and the Kerr-like medium. The first two nonlinear part are similar to the ones appearing in the case of one mode, while the third one is a bilinear connection between the two modes. This bilinear interaction can be used to generate Schrödinger cat states [13] and fulfill quantum teleportation [15].

We assume that the atom and the cavity are initially disentangled, such that the initial state for the total system is of the form

$$|\psi(0)\rangle_{AF} = |\varphi(0)\rangle_A \otimes |\phi(0)\rangle_F, \quad (2)$$

where the initial atomic state is in the superposition of excited state and ground state $|\varphi(0)\rangle_A = \cos(\theta/2)|e\rangle + \sin(\theta/2)|g\rangle$, the cavity is initially in one type of two-mode SU(1,1) coherent states [19]

$$\begin{aligned} |\phi(0)\rangle_F &= |\xi, \frac{1}{2}(1+q)\rangle \\ &= \sum_{n=0}^{\infty} \left(1 - |\xi|^2\right)^{\frac{1+q}{2}} \sqrt{\frac{(n+q)!}{n!q!}} \xi^n |n+q, n\rangle, \quad (3) \end{aligned}$$

which is the eigenstate of the operator $a_1^\dagger a_1 - b_1^\dagger b_1$ with the corresponding eigenvalue $q (= 0, 1, 2...)$. For the case $q = 0$ it is the two-mode squeezed vacuum state. For $q \neq 0$ it is Perelomov coherent state, which is obtained by the action of the two-mode squeeze operator on the number state $|q, 0\rangle$ [20].

The total wave function of the combined quantum system is determined by Schrödinger equation with the Hamiltonian given by equation (1) and initial condition (2). At any time $t \geq 0$ the wave function is expressed as

$$|\psi(t)\rangle_{AF} = \sum_{n=0}^{\infty} \left[A_n(t)|e\rangle + B_n(t)|g\rangle\right]_A |n+q, n\rangle_F, \quad (4)$$

where

$$\begin{aligned} A_n(t) = {} & \exp\left[-\frac{1}{2} i (c_{n+1} + c_n) t\right] \\ & \times \left\{ i \sin\frac{\theta}{2} \sin\left(\frac{1}{2}\sqrt{\Delta_n} t\right) f_{n+1} \frac{(c_{n+1} - c_n)^2 - \Delta_n}{2g\sqrt{(n+1)(n+1+q)\Delta_n}} \right. \\ & \left. + \cos\frac{\theta}{2} f_n \left[\cos\left(\frac{1}{2}\sqrt{\Delta_n} t\right) + i \frac{c_{n+1} - c_n}{\sqrt{\Delta_n}} \sin\left(\frac{1}{2}\sqrt{\Delta_n} t\right)\right] \right\}, \\ B_{n+1}(t) = {} & \exp\left[-\frac{1}{2} i (c_{n+1} + c_n) t\right] \\ & \times \left\{ \sin\frac{\theta}{2} f_{n+1} \left[\cos\left(\frac{1}{2}\sqrt{\Delta_n} t\right) - i \frac{c_{n+1} - c_n}{\sqrt{\Delta_n}} \sin\left(\frac{1}{2}\sqrt{\Delta_n} t\right)\right] \right. \\ & \left. - 2i \cos\frac{\theta}{2} f_n \frac{g\sqrt{(n+1)(n+1+q)}}{\sqrt{\Delta_n}} \sin\left(\frac{1}{2}\sqrt{\Delta_n} t\right) \right\}, \\ B_0(t) = {} & f_0 \sin\left(\frac{\theta}{2}\right) \exp(-i c_0 t), \end{aligned} \quad (5)$$

with

$$\begin{aligned} c_n &= \chi_1 (n+q)(n+q-1) + \chi_2 (n-1) n \\ &\quad + 2\sqrt{\chi_1\chi_2} (n+q) n, \\ \Delta_n &= (c_{n+1} - c_n)^2 + 4g^2 (n+1)(n+1+q), \\ f_n &= \left(1 - |\xi|^2\right)^{\dfrac{1+q}{2}} \sqrt{\frac{(n+q)!}{n!q!}} \xi^n. \end{aligned} \quad (6)$$

Taken as a whole, the system in an overall pure state constitutes a tripartite quantum system including an atom and two modes in a Hilbert space with tensor product structure $2 \otimes \infty \otimes \infty$. If we take the atom as one subsystem with two dimensions, two-mode cavity as another subsystem with infinite dimensions, then the final state of the whole system with tensor product structure $2 \otimes \infty$, can be rewritten as the form of Schmidt decomposition

$$|\psi(t)\rangle_{AF} = \sqrt{\Pi^+}|\varphi_+\rangle_A|\phi_+\rangle_F + \sqrt{\Pi^-}|\varphi_-\rangle_A|\phi_-\rangle_F, \quad (7)$$

here $|\varphi_\pm\rangle_A(|\phi_\pm\rangle_F)$ are the eigenvectors for density matrix of the atom $\rho_A = \mathrm{Tr}_F\rho_{AF}$ (cavity $\rho_F = \mathrm{Tr}_A\rho_{AF}$), $\Pi^\pm$ are the corresponding eigenvalues.

3 Entanglement distribution in the system

We first review on the measures of entanglement. There are a number of measures to quantify the amount of entanglement [4,17,21], and each measure has its limits. For a pure state ρ_{AB} with two subsystems A and B, the von Neumann entropy of the reduced density operators

$$S(\rho_{AB}) = -\mathrm{Tr}(\rho_i \log_2 \rho_i) \quad (i = A, B), \quad (8)$$

is a good measure of entanglement. It takes from zero for a disentangled pure state to one for a maximally entangled one. However, for mixed states ρ_{AB} von Neumann entropy fails, because it can not distinguish classical and quantum mechanical correlations. Another good measurement is called entanglement of formation (EOF) [21], which is defined as $E(\rho) = \min_i p_i S(|\psi_i\rangle\langle\psi_i|)$, where the minimum is taken over all possible decompositions $\rho = \sum_i p_i|\psi_i\rangle\langle\psi_i|$. For some special case, i.e., $2\otimes2$ states, the EOF is obtained analytically in terms of concurrence [4], but in general it is difficult to get an analytical result. In this paper the amount of entanglement is measured by quantum relative entropy of entanglement defined as [17]

$$E(\rho) = \min_{\sigma\in\mathcal{D}} S(\rho||\sigma), \quad (9)$$

where $S(\rho||\sigma) = \mathrm{Tr}\left[\rho\left(\log_2\rho - \log_2\sigma\right)\right]$, is quantum relative entropy, the minimum is taken over $\mathcal{D}$, the set of all disentangled states. Just like EOF, relative entropy of entanglement reduces to the von Neumann entropy of either subsystem for the bipartite pure states, but hard to calculate for general mixed states except some special cases. Now we give a theorem suitable for us to calculate the entanglement between two modes, which states as follows [22].

If a bipartite quantum state is given by the form

$$\rho = \sum a_{n_1,n_2}\left|\alpha_{n_1}\beta_{n_1}\right\rangle\left\langle\alpha_{n_2}\beta_{n_2}\right|, \quad (10)$$

then relative entropy of entanglement for the state is given by

$$E(\rho) = -\sum a_{n,n}\log_2 a_{n,n} - S(\rho), \quad (11)$$

and the disentangled state that minimizes the relative entropy is $\sigma = \sum_n a_{n,n}|\alpha_n\beta_n\rangle\langle\alpha_n\beta_n|$, $|\alpha_n\rangle$ and $|\beta_n\rangle$ are orthonormal states of each subsystem, $S(\rho) = -\mathrm{Tr}(\rho\log_2\rho)$ is the von Neumann entropy mentioned above.

Our system involves four nonequivalent partitions of entanglement: (i) atom-two modes entanglement, E_{A,F_1F_2}; (ii) mode-(atom+mode) entanglement, E_{F_i,AF_j} $(i,j = 1,2,\ i \neq j)$; (iii) mode-mode entanglement, E_{F_1,F_2}; and (iv) mode-atom entanglement, E_{A,F_i} $(i = 1,2)$. It is noted that the mode-atom entanglement is always zero, since the density matrix of atom and one mode is written as the form of their tensor products $\rho_{AF_{1(2)}} = \sum_n p_n(|\varphi_n\rangle\langle\varphi_n|)_A \otimes (|n+q(n)\rangle\langle n+q(n)|)_{F_{1(2)}}$, where $|\varphi_n\rangle_A = \cos(\alpha_n)|e\rangle + \sin(\alpha_n)\exp(i\gamma_n)|g\rangle$ denotes the atomic state, and $\sum_n p_n = 1$.

In the next section we will investigate the entanglements of the cases (i), (ii) and (iii), respectively.

3.1 Atom-two modes and mode-(atom+mode) entanglements

Under the condition that the system is in an overall pure state, atom-two modes entanglement and mode-(mode+atom) entanglement can be calculated by applying equation (8), respectively,

$$E_{A,F_1F_2} = -\Pi^+\log_2\Pi^+ - \Pi^-\log_2\Pi^-, \quad (12)$$

and

$$\begin{aligned} E_{F_1,AF_2} &= S(\rho_{F_1}), \\ E_{F_2,AF_1} &= E_{F_1,AF_2}, \end{aligned} \quad (13)$$

where $\Pi^\pm$, the eigenvalues of the density matrix ρ_A, are given by

$$\begin{aligned} \Pi^+ &= \frac{1}{2}\left\{1 + \left[1 + 4\left(\Big|\sum_n A_n(t)B_n^*(t)\Big|^2 \right.\right.\right. \\ &\quad \left.\left.\left. + \left(\sum_n \Big|A_n(t)\Big|^2\right)^2 - \sum_n\Big|A_n(t)\Big|^2\right)\right]^{1/2}\right\} \\ \Pi^- &= 1 - \Pi^+. \end{aligned} \quad (14)$$

The time evolutions of the entanglements for different conditions, are shown in Figures 1–3. For clarity, Two cases are considered with respect to the Kerr-like medium.

Case 1. Without Kerr-like medium

Two-mode squeezed vacuum state cavity field. Figure 1a is plotted for the case where $q = 0$, $\chi_1 = \chi_2 = 0$, $\theta = 0$, $\xi = 0.6$. We can see that: (1) both entanglements evolve periodically with the period π; (2) atom-two modes entanglement is roughly in phase with mode-(atom+mode) entanglement. In particular, when $gt = t_1 \equiv (2m+1)\pi/2$ $(m = 0,1,2,...)$ both get the minimum values; at $gt = t_2 \equiv m\pi$ the mode-(atom+mode) entanglement obtains the maximal value, while the atom-two modes entanglement sharply decreases to the minimum value zero.

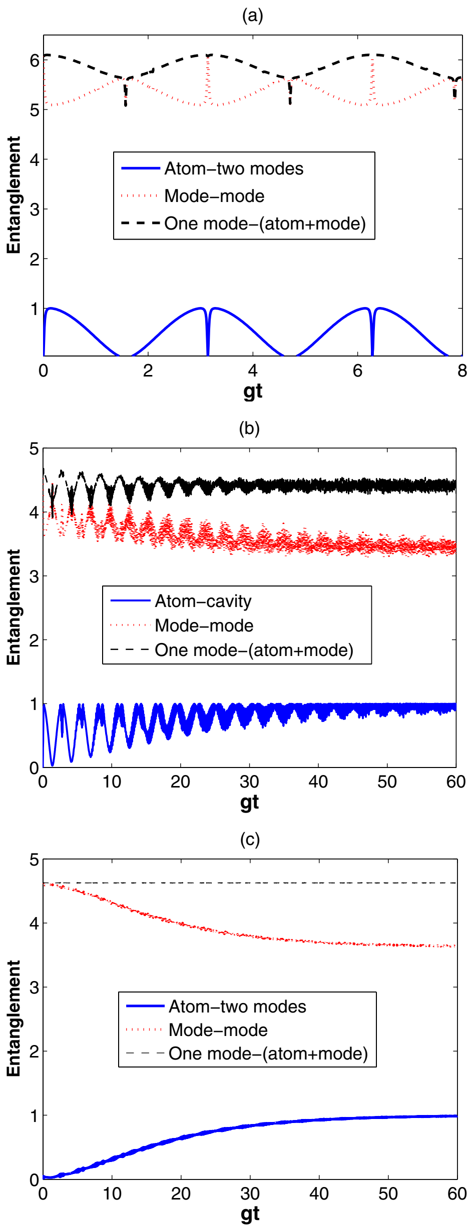

Fig. 1. Entanglement distribution among different subsystems without Kerr-like medium and $\xi = 0.6$. Solid curve (blue), atom-two modes entanglement E_{A,F_1F_2}; dotted curve (red), mode-mode entanglement E_{F_i,F_j}; dashed curve (black), mode-(atom+mode) entanglement E_{F_i,AF_j}. (a) Two-mode squeezed vacuum state field, excited atomic state. (b) Two-mode Perelomov coherent state field, excited atomic state. (c) Two-mode Perelomov coherent state field, $(|e\rangle+|g\rangle)/\sqrt{2}$ atomic state. A color version of the figures is available in electronic form at `http://www.eurphysj.org`.

In fact, this can seen from equations (4–6). At time $gt = t_1$, it is easy to find that the state of the atom and two-mode cavity field is

$$|\psi(t_1)\rangle_{AF} = |\varphi(t_1)\rangle_A \otimes |\phi(t_1)\rangle_F, \tag{15}$$

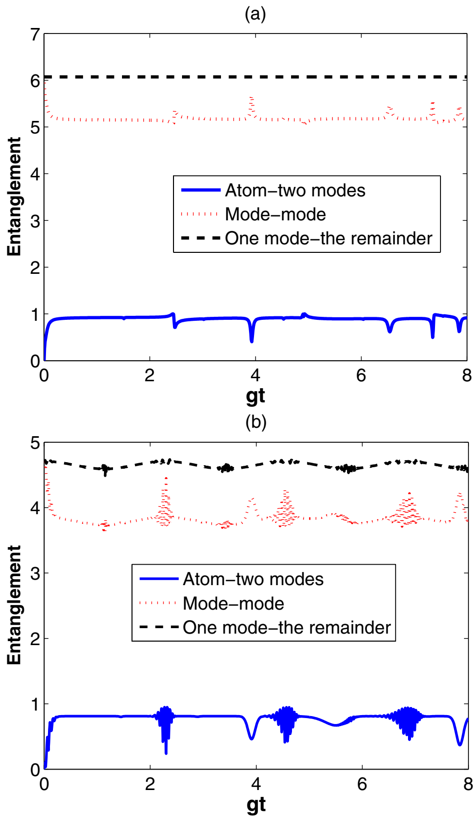

Fig. 2. Entanglement distribution among different subsystems in the presence of Kerr-like medium with $\chi_1 = \chi_2 = 0.2$ and $\xi = 0.6$. Solid curve (blue), atom-two modes entanglement E_{A,F_1F_2}; dotted curve (red), mode-mode entanglement E_{F_i,F_j}; dashed curve (black), mode-(atom+mode) entanglement E_{F_i,AF_j}. (a) Two-mode squeezed vacuum state field, excited atomic state. (b) Two-mode Perelomov coherent state field, excited atomic state. A color version of the figures is available in electronic form at `http://www.europhysj.org`.

here the state of atom and cavity field are given by, respectively,

$$\begin{aligned}
|\varphi(t_1)\rangle_A &= \left[1+|\xi|^2\right]^{-\frac{1}{2}}(-i\xi|e\rangle+|g\rangle),\\
|\phi(t_1)\rangle_F &= \sqrt{1-|\xi|^4}\sum_{n=0}^{\infty}(-1)^n\xi^{2n}\\
&\quad\times\left(\sin\frac{\theta}{2}|2n,2n\rangle - i\cos\frac{\theta}{2}|2n+1,2n+1\rangle\right).
\end{aligned}\tag{16}$$

Therefore the atom and two modes are disentangled, and the atomic information encoded in θ is transferred to the cavity field. At time $gt = t_2$, they are also disentangled, the states for the subsystems are given by

$$\begin{aligned}
|\varphi(t_2)\rangle_A &= \sin\frac{\theta}{2}|g\rangle - \cos\frac{\theta}{2}|e\rangle,\\
|\,\phi(t_2)\rangle_F &= \begin{cases}\sum_n f_n(-1)^n\,|\,n,n\rangle & \text{for } m = \text{odd}\\ \sum_n f_n\,|\,n,n\rangle & \text{for } m = \text{even}.\end{cases}
\end{aligned}\tag{17}$$

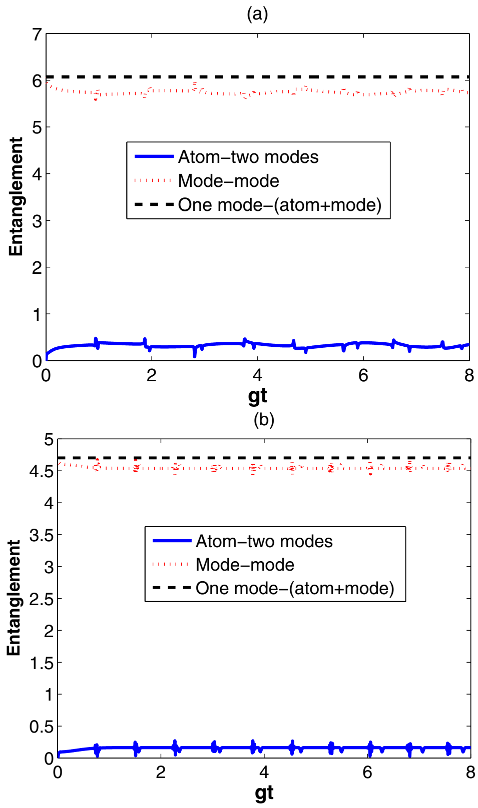

Fig. 3. Same as Figure 2, except for $\chi_1 = \chi_2 = 1$. A color version of the figures is available in electronic form at `http://www.europhysj.org`.

Contrasted to the case of t_1, atomic information is not transferred to cavity field, the atomic state is varied by phase π. It is noted that equations (16) and (17) give the different information about the subsystems, even the atom and cavity field are disentangled in both cases. According to equations (16, 17), mode-(atom+mode) entanglements at t_1 and t_2 are calculated as, respectively

$$\begin{aligned} E_{F_i,AF_j}(t_1) &= -\log_2\left(1-|\xi|^4\right) - \frac{4|\xi|^4}{1-|\xi|^4}\log_2|\xi| \\ &\quad -2\sin^2\frac{\theta}{2}\log_2\left|\sin\frac{\theta}{2}\right| - 2\cos^2\frac{\theta}{2}\log_2\left|\cos\frac{\theta}{2}\right|, \\ E_{F_i,AF_j}(t_2) &= -\frac{1}{1-|\xi|^2}\log_2\left(1-|\xi|^2\right) \\ &\quad -\frac{|\xi|^2}{1-|\xi|^2}\log_2\frac{|\xi|^2}{1-|\xi|^2}, \end{aligned} \tag{18}$$

It is easy to see that the latter is greater than the former for $\theta = 0$, $\xi = 0.6$.

Two-mode Perelomov coherent state cavity field. The entanglements are no longer periodic due to the non-symmetry of the two modes, the atom and the cavity are always entangled during the interaction. The entanglements oscillate, but the amplitude is gradually weakened with the time. As a result the atom-two modes entanglement gradually increases, while mode-(atom+mode) entanglement decreases a little as time goes on. Finally $E_{A,F_1F_2} \approx 1$, the atom and two modes are almost maximally entangled. Therefore we can get a maximally entangled state of atom and cavity if given enough long time. It is interesting to note that if the atomic state is initially in or approaches the state $|s\rangle = (|e\rangle + |g\rangle)/\sqrt{2}$ the oscillations of entanglements are reduced greatly, the entanglement of mode-(atom+mode) is approximately constant. This is because the state $|s\rangle$ is a dressed-state(stationary state), which shows no dynamic evolution. The small vibrations of the quantum relative entropy in the case come from the contribution of the interaction between the atom and cavity.

Case 2. In the presence of Kerr-like medium

Now we turn to the effects of Kerr-like medium. To visualize the effects of the Kerr-like medium on the entanglement, we set different values of χ_i/g with all the other parameters with the same values as in Figure 1. The results are presented in Figures 2 and 3 for weak and strong nonlinear interactions of the Kerr-like medium with the field modes, respectively.

Comparing with Figure 1a and Figures 2a, 3a, we find that when the initial state for cavity is two-mode squeezed state, the nonlinear interaction of Kerr-like medium leads to the nonperiodicity of entanglement evolution, the atom and cavity are always entangled.

From Figure 1b and Figures 2b, 3b, where the cavity is initially in the Perelomov coherent state for the two-mode field, it follows that mode-(atom+mode) entanglement can not gradually increase as time goes on, but reach a stable value, in the vicinity of which it oscillates with a tiny amplitude, and cannot reach the maximum value 1 even given long time evolution. For the entanglement between one mode and remainder, it is almost unchanged. Those results imply that the nonlinear interaction prevents the atom and two modes from disentangling.

We also show that the intensity of the nonlinear interaction has significantly influence on the entanglement distribution. Contrasted to the strong nonlinear interaction, the weak nonlinear interaction increases the minimum value of atom-two modes entanglement and the sustaining time of the maximum atom-two modes entanglement, as a result the atom and field maintain strongly entangled. With the increase of the nonlinearly coupling strength of the Kerr-like medium with the field modes, the entanglement between the atom and field reduces. The results are in accord with those given in [16]. For the mode-(atom+mode) entanglement, it is approximately a constant in the presence of strong nonlinear interaction of the Kerr-like medium with the field modes. We will give the reason in the next section.

3.2 Mode-mode entanglement

The final bipartite partition of entanglement is the one between two modes F_1 and F_2. The system F_1F_2 is obtained by tracing over the atom. Different from the pure state bipartite systems such as $A-(F_1F_2)$, $F_i-(AF_j)$, the two modes system F_1-F_2 is generally a mixed state in a Hilbert space with tensor product structure $\infty\otimes\infty$,

$$\rho_{F_1F_2}=\sum_{k,n=0}^{\infty}a_{k,n}|k+q,k\rangle\langle n+q,n|, \tag{19}$$

where $a_{k,n}=A_kA_n^*+B_kB_n^*$. It can be seen that the density matrix of equation (19) takes the form given by equation (10). Therefore it is possible to compute numerically the mode-mode entanglement E_{F_1,F_2}

$$E_{F_1,F_2}=-\sum_{n=0}^{\infty}a_{n,n}\log_2 a_{n,n}-S(\rho_{F_1F_2}). \tag{20}$$

The middle (black) curves in Figures 1, 2 and 3 give the time evolutions of the mode-mode entanglement for the different initial conditions. The two modes coupled to a single atom can change the entanglement between them, even when they do not interact directly. From all figures, we find that the mode-mode entanglement is always out of phase with atom-two modes entanglement, that is, the mode-mode entanglement decreases (increases) at the time when the atom-two modes entanglement increases (decreases). The results can be interpreted as follows. The two-mode cavity is considered as a system, and the single atom as the bath of the system. The interaction between cavity field and atom leads to the entanglement between them, in turn, the entanglement between them impairs that between two modes, even the two modes do not interact directly. If the atom is regarded as the environment of two-mode cavity, the decoherence of two modes is induced by the environment. In order to see the effect of Kerr-like medium, we consider the following two cases.

Case 1. No Kerr-like medium

We first consider the case of no Kerr-like medium. For cavity initially being in two-mode squeezing state, the evolution of mode-mode entanglement is periodic. At time $gt=t_1$ and $gt=t_2$, the states of the two modes are given by equations (16, 17), the corresponding entanglements read

$$\begin{aligned}E_{F_1,F_2}(t_1)&=E_{F_i,AF_j}(t_1),\\ E_{F_1,F_2}(t_2)&=E_{F_i,AF_j}(t_2).\end{aligned} \tag{21}$$

It is noted that mode-mode entanglement and atom-two modes entanglement can not simultaneously archive the maximum values. For two-mode Perelomov coherent state of the initial field, i.e., $q\neq 0$, the two-mode entanglement is not periodic, but gradually decreases with the time. Given long time scale, two modes is minimally entangled.

Case 2. Kerr-like medium

Figures 2 and 3 show the influence of Kerr-like medium on the mode-mode entanglement. The mode-mode entanglement does not exhibit the periodicity ($q=0$) or the tendency of gradual reduction with time ($q\neq 0$). After some decrease, the two modes entanglement oscillates little in the vicinity of some value and two modes possess large amount of entanglement during the time evolution due to the modulation of the nonlinear interaction of Kerr-like medium with the two modes. If the coupling between Kerr-like medium and two-mode field is much stronger than the coupling between the single atom and the two-mode field, i.e., $\chi_i/g\gg 1$, the entanglement between two modes retains unchanged, and the coherence of two modes keeps well. This is because that, in this situation the atom and two-mode field are approximately decoupled, thus all partitions of entanglements are unchanged.

4 Conclusion

In summary, we investigate the entanglement distribution in a quantum optics system with the structure $2\otimes\infty\otimes\infty$. The entanglements between different two subsystems are numerically calculated. By the numerical results, we analyze the time evolutions of entanglements in different cases. (1) In the absence of Kerr-like medium, if the initial cavity field state is two-mode squeezed state, the evolutions for different partitions of entanglements are periodic, and the mode-mode entanglement and mode-(atom+mode) entanglement are equal when the atom and the cavity are disentangled. If the initial sate of field is Perelomov coherent state, the atom-two modes entanglement is out of phase with mode-mode entanglement. The atom and two modes become more and more strongly entangled as time goes on, while the two modes get more and more weakly entangled. (2) In the presence of Kerr-like medium, the time evolutions for entanglements are significantly different, the entanglement between atom and field can not get more entangled, the two modes maintain large amount of entanglement, and the mode-(atom+mode) entanglement changes little during the evolution. When the coupling between the Kerr-like medium and field is much stronger than that between the atom and field, all entanglement between two subsystems are approximately unchanged. (3) The paper also gives the simple interpretation of what are mentioned above.

The paper does not consider decoherence effects associated with atomic spontaneous emission and with cavity decay. The state of the system is described by its master equation, and the explicit state of the whole system, usually a mixed state, is hardly given in the general case. In the presence of decay, another problem is whether the entanglement is analytically expressed in terms of quantum relative entropy. In fact any mixed system can be purified a pure one by adding another system, therefore one can research entanglement distribution in the larger space of the initial system plus the added system, which is beyond our scope.

We thank the referees for their comments and valuable suggestions. This work was funded by National Fundamental Research Program (2001CB309300), the Innovation funds from Chinese Academy of Science, National Natural Science Foundation of China under Grant No. 60121503.

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